

Fermion Mass Hierarchies and Flavour Mixing from a Minimal Discrete Symmetry

Ferruccio Feruglio ¹ and Yin Lin ²

Dipartimento di Fisica ‘G. Galilei’, Università di Padova
INFN, Sezione di Padova, Via Marzolo 8, I-35131 Padua, Italy

Abstract

We construct a simple model of fermion masses based on a spontaneously broken $S_3 \times Z_3$ flavour group. At the leading order, in the neutrino sector S_3 is broken down to a $\nu_\mu - \nu_\tau$ parity subgroup that enforces a maximal atmospheric mixing angle and a vanishing θ_{13} . In the charged lepton sector the $\nu_\mu - \nu_\tau$ parity is maximally broken and the resulting mass matrix is nearly diagonal. The charged lepton mass hierarchy is automatically reproduced by the S_3 symmetry breaking parameter alone. A careful analysis shows that, after the inclusion of all relevant subleading effects, the model predicts $\theta_{23} = \pi/4 + O(\lambda_c^2)$ and $\theta_{13} = O(\lambda_c^2)$, λ_c denoting the Cabibbo angle. A simple extension to the quark sector is also illustrated, where the mass spectrum and the mixing angles are naturally reproduced, with the exception of the mixing angle between the first two generations, that requires a small accidental enhancement.

¹e-mail address: feruglio@pd.infn.it

²e-mail address: Lin@pd.infn.it

1 Introduction

On the eve of the tenth anniversary of the SuperKamiokande (SK) data on atmospheric neutrinos, that shook the whole field and gave rise to a decade of incredible excitement and activity, neutrino oscillation parameters are known to a sufficiently high precision to make desirable a theoretical description going beyond the mere fitting procedure. In particular the leptonic mixing pattern, so different from the one in the quark sector, provides a non-trivial theoretical challenge. The present data [1]:

$$\theta_{12} = (34.5 \pm 1.4)^0 \quad , \quad \theta_{23} = (42.3^{+5.1}_{-3.3})^0 \quad , \quad \theta_{13} = (0.0^{+7.9}_{-0.0})^0 \quad , \quad (1)$$

are fully compatible with the so-called tri-bimaximal (TB) mixing pattern, where

$$\sin^2 \theta_{12} = \frac{1}{3} \quad (\theta_{12} = 35.3^0) \quad , \quad \sin^2 \theta_{23} = \frac{1}{2} \quad , \quad \sin^2 \theta_{13} = 0 \quad . \quad (2)$$

Several theoretical mechanisms leading to a nearly TB mixing have been suggested in the last years [2, 3, 4, 5]. The TB mixing has the advantage of correctly describing the solar mixing angle, which, at present, is the most precisely known. Indeed, its 1σ error, 1.4 degrees corresponds to less than λ_c^2 radians, where $\lambda_c \approx 0.22$ denotes the Cabibbo angle.

Reaching a similar sensitivity on θ_{23} and θ_{13} will require some more years of work, but it is a remarkable feature that the central values of these angles remained surprisingly stable in the last ten years. The value of θ_{23} quoted above is largely dominated by the SK data. It is however noticeable that the independent determinations of θ_{23} by MACRO [6], K2K [7], MINOS [8] and SK [9], analyzed in a two-flavour framework, all select $\sin^2 2\theta_{23} = 1$ as best fit value. By removing the boundary $\sin^2 2\theta_{23} \leq 1$, K2K, MINOS and SK prefer $\sin^2 2\theta_{23}$ slightly outside the physical region. Notice that the value of θ_{23} extracted from three-flavour global fits, quoted in eq. (1), is slightly non-maximal. Such an effect becomes manifest when we move from the two-flavour analysis to the three-flavour one [10]. In the last case, the dependence on Δm_{12}^2 is included in the analysis of atmospheric neutrino data and the deviation from maximality is due to the presence of a small excess of events in the sub-GeV electron sample of SK, which the two-flavour analysis cannot completely account for. At the moment such a deviation is not statistically significant, but future, more precise data might confirm that the maximality of θ_{23} is violated at the λ_c^2 level. The upper bound on θ_{13} is dominated by the CHOOZ data [11], though the preference for a small θ_{13} is also present in the solar and the atmospheric data samples. Moreover, the recent significant error reduction on $|\Delta m_{23}^2|$ by MINOS has also sharpened the CHOOZ bound on θ_{13} , which depends on $|\Delta m_{23}^2|$. From the theory point of view, maximal and vanishing mixing angles are special and many theoretical efforts have been devoted to explain how $\theta_{23} = \pi/4$ and/or $\theta_{13} = 0$ can be generated.

As a matter of fact, in the so-called flavour basis, the most general neutrino mass matrix giving rise to $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ displays a $\nu_\mu - \nu_\tau$ parity symmetry [12]. Such a parity symmetry, when extended to the whole theory, is expected to be broken in the charged lepton sector by the large hierarchy $m_\mu/m_\tau \ll 1$. In a realistic model the breaking effects, responsible for non-vanishing $\theta_{23} - \pi/4$ and θ_{13} , should have only a small impact on the neutrino sector. Assuming diagonal charged leptons, textures supported

by the $\nu_\mu - \nu_\tau$ symmetry, including breaking effects, have been widely studied in the literature ¹ [14, 15, 16]. Special attention have been paid to the effect of small $\nu_\mu - \nu_\tau$ symmetry breaking terms coming from the neutrino sector on the values $\cos 2\theta_{23}$ and $\sin^2 \theta_{13}$. Depending on the type of neutrino mass hierarchy, correlations among small quantities such as $\sin^2 \theta_{13}$, $\cos 2\theta_{23}$ and $R = \Delta m_{sol}^2 / \Delta m_{atm}^2$ have been investigated. In particular, in simple frameworks, it turns out that $\sin^2 \theta_{13}$ is strongly suppressed by R for normal hierarchy [14]. Since the Dirac phase is absent in the $\nu_\mu - \nu_\tau$ symmetric limit, we expect that deviations of $\theta_{23} - \pi/4$ and θ_{13} from zero are sensitive to CP phases (including Majorana phases) [14]. Moreover, the breaking of the $\nu_\mu - \nu_\tau$ symmetry can be driven entirely by the introduction of CP violating phases [16]. In this case strict correlations between the CP violation and the broken $\nu_\mu - \nu_\tau$ symmetry can be established ². However, in a general framework where U_l , the contribution to lepton mixing from the charged leptons, is only approximately equal to the unit matrix, many of these correlations are relaxed. For this reason, it is particularly instructive to explore models of fermion masses providing a consistent and unified picture of the neutrino sector and of the hierarchy among charged lepton masses. Only in such a context it will be possible to keep under control all possible breaking effects of the $\nu_\mu - \nu_\tau$ symmetry and to achieve a realistic next-to-leading order prediction for $\cos 2\theta_{23}$ and $\sin^2 \theta_{13}$.

Nowadays promising candidates for a unified picture of fermion mass hierarchies and flavour mixing are the models based on spontaneously broken flavour symmetries. The mixing angles, in particular the leptonic ones, are best understood by a mechanism of vacuum misalignment occurring in theories with non-abelian flavour symmetries ³. In the various fermion sectors of the theory (up quarks, down quarks, charged leptons and neutrinos) the symmetry is broken along different directions in flavour space and the corresponding diagonalizations require misaligned unitary transformations, which end up in the desired mixing pattern. Such breaking schemes are easy to arrange in SUSY models based on small discrete symmetry groups, where the discussion of vacuum alignment is particularly simple and transparent. Also the fermion mass hierarchies can be achieved via spontaneous breaking of the flavour symmetry. However, in most cases, a separate component of the flavour group is exploited to this purpose. Quite frequently the flavour group is of the type $D \times U(1)_{FN}$ where D is a discrete component that controls the mixing angles and $U(1)_{FN}$ is an abelian continuous symmetry that describes the mass hierarchy, along the lines of the original Froggatt-Nielsen proposal [19]. It would be desirable to have a more economical model where the same flavon fields producing the mixing pattern via VEV misalignment are also responsible for the mass hierarchies. Models of this type

¹The exchange symmetry between the second and the third generations has also been adopted as a texture symmetry not only for neutrinos but for all fermions [13]. Both lepton and quark mixing angles can be reproduced and there are enough parameters to fit the fermion masses, without however explaining their hierarchies.

²CP violating phases associated to $\nu_\mu - \nu_\tau$ breaking effects are also relevant in leptogenesis. In simple realizations of the seesaw mechanism, an exact $\nu_\mu - \nu_\tau$ symmetry in the Dirac mass matrix and in the right-handed neutrino mass matrix implies a vanishing primordial lepton asymmetry. A successful leptogenesis requires appropriate extensions of this scheme [17].

³Spontaneously broken discrete symmetries with preserved sub-groups can play also an important role in explaining the Cabibbo angle, see for instance [18].

based on the gauged flavour groups $SU(3)$ and $SO(3)$ exist in the literature [20]. In these models the charged fermion mass hierarchies are obtained via a flamboyant flavour symmetry breaking sector and a particular choice of the messenger scales. It would be interesting to identify a kind of minimal flavour group with few flavon fields, able to provide a decent description of the main features of the fermion mass spectrum, including the approximate vanishing of θ_{13} and of $\theta_{23} - \pi/4$.

In this paper we illustrate a model for lepton masses based on the small discrete group $S_3 \times Z_3$. The non-abelian factor S_3 is spontaneously broken by a special vacuum misalignment. This guarantees the relations $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ at the lowest order in the expansion parameters $\langle\varphi\rangle/\Lambda \ll 1$ describing the symmetry breaking of S_3 . The same expansion also provides the required suppression factors that “explain” the observed hierarchies among charged lepton masses. There is no need of additional flavons or of additional group factors to describe, at the level of orders of magnitude, all lepton masses. The Z_3 factor remains unbroken to forbid unwanted couplings between different fermion sectors. It is also of great experimental interest to establish at which order in $\langle\varphi\rangle/\Lambda \ll 1$ the leading order results for θ_{23} and θ_{13} are potentially violated. After a detail analysis of all breaking effects, we get:

$$\theta_{23} = \frac{\pi}{4} + O(\lambda_c^2) \quad , \quad \theta_{13} = O(\lambda_c^2) \quad , \quad (3)$$

where λ_c is the Cabibbo angle. The structure of the model is very simple. The main feature is the spontaneous breaking of S_3 by two sets of flavon fields, φ_e and (φ_ν, ξ) . Thanks to a vacuum alignment property, (φ_ν, ξ) breaks S_3 down to a $\nu_\mu - \nu_\tau$ parity subgroup, while φ_e breaks S_3 completely. In particular the $\nu_\mu - \nu_\tau$ parity is maximally broken by φ_e . These two breaking patterns are selectively supplied to the Yukawa couplings of the theory. At the leading order of the expansion in VEV/Λ , Λ being the cutoff of the theory, φ_e affects only the charged lepton mass matrix, which is nearly diagonal, while (φ_ν, ξ) controls the neutrino mass matrix, which remains invariant under the $\nu_\mu - \nu_\tau$ exchange, thus guaranteeing $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. Such a scheme is particularly economic in terms of new fields and new symmetries. A distinguished feature of our model is that the hierarchy of the masses in the charged lepton sector is also controlled by the S_3 breaking and m_τ, m_μ, m_e get their first non-vanishing contribution at the order $\langle\varphi_e\rangle/\Lambda, (\langle\varphi_e\rangle/\Lambda)^2, (\langle\varphi_e\rangle/\Lambda)^3$, respectively. This allows to estimate $\langle\varphi_e\rangle/\Lambda \approx \lambda_c^2$. We also analyze all the effects that can modify $\theta_{23} = \pi/4$ and $\theta_{13} = 0$. We discuss the sub-leading operators and we carefully minimize the scalar potential of the theory by including the relevant sub-leading corrections in order to estimate the deviations of the VEVs from the leading order alignment. The leading order alignment is not spoiled provided $\langle\varphi_\nu\rangle, \langle\xi\rangle \leq \langle\varphi_e\rangle \approx \lambda_c^2 \Lambda$. In this regime the expected corrections to $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ are of order λ_c^2 . Finally, we extend our construction to the quark sector. The down quark mass hierarchy is reproduced by the breaking of S_3 , while the more pronounced hierarchy in the up quark sector requires an additional, spontaneously broken, Z'_3 factor in the flavour group. The quark mixing angle are correctly reproduced, with the exception of the Cabibbo angle, which requires an accidental enhancement by a factor 3-4.

2 A problem with the $\nu_\mu - \nu_\tau$ symmetry

The most general pattern of lepton masses that automatically leads to $\theta_{13} = \theta_{23} - \pi/4 = 0$ is characterized by a $\nu_\mu - \nu_\tau$ exchange symmetry, as we briefly recall now. After breaking of the total lepton number (assumed hereafter) and of the electroweak symmetry, in a two-component spinor notation the lepton mass terms of the lagrangian read:

$$\mathcal{L} = -e^c m_l e - \frac{1}{2} \nu m_\nu \nu \quad (4)$$

where m_l and m_ν are the charged lepton mass matrix and the effective neutrino mass matrix. An arbitrary change of basis in the generation space, realized by means of unitary transformations Ω_{e^c} and Ω_l acting on e^c and $l = (\nu, e)$ respectively, modifies the form of m_l and m_ν , but does not change the physics, in particular the charged lepton masses m_e, m_μ, m_τ , the neutrino masses m_1, m_2, m_3 and the mixing matrix U_{PMNS} . We can exploit this freedom to render diagonal the charged lepton mass matrix m_l :

$$m'_l = \text{diag}(m_e, m_\mu, m_\tau), \quad m'_\nu = U_{PMNS}^* \text{diag}(m_1, m_2, m_3) U_{PMNS}^\dagger \quad (5)$$

In this basis, the effective neutrino mass matrix is completely determined by the measurable quantities m_i and U_{PMNS} . Indeed, any complex symmetric 3×3 matrix has 9 real parameters as the number of parameters in m_i and U_{PMNS} . In the limit where both θ_{13} and $\theta_{23} - \pi/4$ vanish, the PMNS mixing matrix becomes (apart from sign convention redefinitions and majorana phases):

$$U_{PMNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \quad (6)$$

where we are left with the only dependence on the angle θ_{12} , through $c_{12} \equiv \cos \theta_{12}$ and $s_{12} \equiv \sin \theta_{12}$.

In the preferred basis of eq. (5), the conditions $\theta_{13} = \theta_{23} - \pi/4 = 0$ correspond to the requirement that the effective neutrino mass matrix m_ν has the form:

$$m'_\nu = \begin{pmatrix} c - 2b & d & d \\ d & b + a & b - a \\ d & b - a & b + a \end{pmatrix}, \quad (7)$$

where a, b, c, d are functions of $m_1, m_2, m_3, \theta_{12}$:

$$\begin{aligned} a &= 1/2 m_3 \\ b &= 1/2 (m_1 s_{12}^2 + m_2 c_{12}^2) \\ c &= m_1 + m_2 \\ d &= 1/\sqrt{2} (m_2 - m_1) c_{12} s_{12} \end{aligned} \quad (8)$$

All equivalent patterns of lepton mass matrices giving rise to $\theta_{13} = \theta_{23} - \pi/4 = 0$, are obtained from m'_l diagonal and m'_ν of eq. (7), by means of $U(3)_{e^c} \times U(3)_l$ transformations.

However, in the flavour basis, where eq. (5) holds, it is easier to verify that m'_ν exhibits an exact $\nu_\mu - \nu_\tau$ exchange symmetry. This $\nu_\mu - \nu_\tau$ symmetry cannot be naively extended to the whole lepton sector. Since ν_μ and ν_τ are members of the $SU(2)_L$ doublets (ν_μ, μ) and (ν_τ, τ) , in such a naive extension the theory would also be symmetric under the exchange of the left-handed charged leptons μ, τ . But this property is difficult to reconcile with the large mass hierarchy $m_\mu \ll m_\tau$.

In some proposals, several Higgs doublets and abelian discrete symmetries of Z_N type are introduced to keep diagonal the charged leptons [15, 16, 17, 21], but the mass hierarchies are obtained by fine-tuning the Yukawa parameters. Alternatively, we might try to exploit an abelian $U(1)_F$ flavour symmetry to reproduce the observed smallness of m_μ/m_τ . The simplest possible charge assignment compatible with the $\nu_\mu - \nu_\tau$ parity is to give equal charges to the left-handed doublets (ν_μ, μ) and (ν_τ, τ) , and different charges to μ^c and τ^c . However such a choice implies a large contribution to θ_{23} from the charged lepton sector, which would cause large deviations from $\theta_{23} = \pi/4$.

A partial solution to this problem is given by Grimus et al. [22] in a model based on a flavour symmetry $S_3 \times \dots$, where dots denotes additional factors of the flavour group. We shortly review the main features of this model, by adapting it in order to make easier the comparison with our proposal. Left-handed lepton doublets are chosen as $(1 + 2)$ of S_3 : $l_1 = (\nu_e, e)$ is the invariant singlet and $D_l = ((\nu_\mu, \mu), (\nu_\tau, \tau))$ is the S_3 doublet⁴. Right-handed charged leptons e^c, μ^c, τ^c are in a $(1 + 2^*)$ representation: the singlet is e^c and the conjugate doublet is $D_l^c = (\mu^c, \tau^c)$. The neutrino sector explicitly possesses the $\nu_\mu - \nu_\tau$ symmetry, subgroup of S_3 . In the charged lepton sector, three scalar field φ_i ($i = 1, 2, 3$) are introduced: $\varphi_{1,2}$ are singlets of S_3 and φ_3 belongs to $1'$. Thanks to an additional Z_2 symmetry under which only φ_1 and e^c transform non-trivially, φ_1 couples with e^c while φ_2 doesn't. The Yukawa interactions of the charged lepton sector are given by

$$\mathcal{L}_l = \frac{h_d}{\Lambda} [f_1 e^c l_1 \varphi_1 + f_2 (D_l^c D_l) \varphi_2 + f_3 (D_l^c D_l)' \varphi_3 + h.c.] + \dots \quad (9)$$

When the fields φ_1, φ_2 and φ_3 acquire VEVs v_1, v_2 and v_3 respectively, after electroweak symmetry breaking, the charged leptons acquire a diagonal mass matrix with

$$\begin{aligned} m_e &= f_1 v_1 \\ m_\mu &= f_2 v_2 + f_3 v_3 \\ m_\tau &= f_2 v_2 - f_3 v_3 \end{aligned}$$

We see that the non trivial singlet φ_3 breaks the $\mu - \tau$ symmetry in the charged lepton sector. In order to obtain the hierarchy between the masses of μ and τ a further symmetry, K , is needed:

$$K : \quad (\mu^c, \tau^c) \rightarrow (-\mu^c, \tau^c), \quad \varphi_2 \leftrightarrow \varphi_3 \quad (10)$$

The K -symmetry imposes $f_2 = -f_3$ and $v_2 = v_3$ and, when softly broken by $(v_2 - v_3) \ll (v_2 + v_3)$, we get:

$$\frac{m_\mu}{m_\tau} = \frac{v_2 - v_3}{v_2 + v_3} \ll 1. \quad (11)$$

⁴Basic properties of S_3 are reviewed in the next section.

The K -symmetry does not commute with S_3 , this means that the full symmetry group is not simply $S_3 \times K$, but is rather generated by the elements in S_3 and in K and consequently much larger than S_3 . Furthermore, the smallness of m_e is still not explained by this mechanism. In our construction, the problem discussed here is completely solved and the whole hierarchy $m_e \ll m_\mu \ll m_\tau$ can be obtained without fine-tuning and without the need of any ad-hoc symmetry.

3 A model based on the group S_3

We are thus naturally led to the possibility that the $\nu_\mu - \nu_\tau$ symmetry is only a feature of the neutrino sector and it is strongly broken in the charged lepton sector. It is by now well-known how to realize such an hybrid symmetry pattern. We embed the Z_2 parity arising from the $\nu_\mu - \nu_\tau$ exchange in a larger group G . Then we selectively couple charged leptons and neutrinos to two different scalar multiplets of G , φ_e and (φ_ν, ξ) , respectively. Crossed interaction terms of the type $\varphi_e \nu \nu$ and $(\varphi_\nu, \xi) e^c e$ are forbidden by a Z_3 subgroup of G which will remain unbroken. The multiplets φ_e and φ_ν acquire VEVs that break G into different subgroups. The VEV of φ_ν breaks G down to Z_2 , which becomes the residual symmetry in the neutrino sector, while the VEV of φ_e breaks G down to a different subgroup, not containing Z_2 and guaranteeing a hierarchical, quasi diagonal matrix m_l . Here we will make such a construction explicit by taking $G = S_3 \times Z_3$. As we will see the powerful of such a broken flavour symmetry is that it will also produce a hierarchical structure in the charged lepton mass matrices without the need of an additional abelian symmetry.

S_3 is group of permutations of three distinct objects and is the smallest non-abelian symmetry group. Geometrically, S_3 consists of the rotations in three dimensions leaving invariant an equilateral triangle. It has six elements divided into three conjugate classes: the identity, cyclic and anti-cyclic circulations of triangle apices and three exchanges of two of three apices leaving the third fixed. Therefore S_3 contains three irreducible representations, which are all real.

Classes	n	h	1	1'	2
C_1	1	1	1	1	2
C_2	2	3	1	1	-1
C_3	3	2	1	-1	0

Table 1: Character table of S_3 . C_i with $i=1,2,3$, are the three conjugate classes of the group; n the order of a class, i.e. the number of distinct elements contained in a class; h is the order of an element g in a class, i.e. the smallest integer (> 0) for which $g^h = 1$.

The S_3 group has a presentation given by the generators S and T which satisfy the following relations:

$$S^2 = T^3 = (ST)^2 = 1 . \quad (12)$$

The elements of S_3 can be written as products of generators of the group: $1, S, T, ST, TS, T^2$. The even permutations are generated by T : $\{1, T, T^2\}$ and form a Z_3 subgroup. The one-dimensional unitary representations are given by

$$\begin{aligned} 1 \quad S &= 1, & T &= 1 \\ 1' \quad S &= -1, & T &= 1 \end{aligned}$$

This means that S_3 should be broken down to Z_3 by the VEV of a pseudo singlet scalar field. The two-dimensional unitary representation 2 of S_3 is given by:

$$S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}. \quad (13)$$

S corresponds to an interchange Z_2 symmetry of the two components of a doublet field. For this reason, S_3 can be broken down to a Z_2 subgroup by the VEV of a doublet scalar field: $\langle \varphi \rangle = (v, v)$.

The tensor products involving pseudo-singlets are given by $1' \times 1' = 1$ and $1' \times 2 = 2$. While the product of two doublets is given by $2 \times 2 = 2 + 1 + 1'$. From the complex representations of S and T given in Eq. (13), one can explicitly calculate these basic tensor products. Given two doublets $\psi = (\psi_1, \psi_2)^t$ and $\varphi = (\varphi_1, \varphi_2)^t$, it is easy to see that

$$\begin{aligned} \psi_1 \varphi_2 + \psi_2 \varphi_1 &\in 1 \\ \psi_1 \varphi_2 - \psi_2 \varphi_1 &\in 1' \end{aligned} \quad \begin{pmatrix} \psi_2 \varphi_2 \\ \psi_1 \varphi_1 \end{pmatrix} \in 2 \quad (14)$$

The complex conjugate ψ^* belongs to the anti-doublet representation 2^* for which the representation matrices become (S^*, T^*) . Defining

$$\psi' \equiv \sigma_1 \psi^* = \begin{pmatrix} \psi_2^* \\ \psi_1^* \end{pmatrix}$$

and using $\sigma_1 S^* \sigma_1 = S$ and $\sigma_1 T^* \sigma_1 = T$ one can show that ψ' transforms as a doublet of S_3 . Then from Eq. (14) we have

$$\begin{aligned} \psi_1^* \varphi_1 + \psi_2^* \varphi_2 &\in 1 \\ \psi_1^* \varphi_1 - \psi_2^* \varphi_2 &\in 1' \end{aligned} \quad \begin{pmatrix} \psi_1^* \varphi_2 \\ \psi_2^* \varphi_1 \end{pmatrix} \in 2. \quad (15)$$

The form of tensor product, and consequently the resulting fermion mass pattern, depends on the group basis. Other bases are also adopted in the literature. The most important example is the non irreducible permutation basis in which the representation matrices for S_3 are given by the label-changing matrices. A study of the various basis of S_3 (observe that by a change of basis, it is possible to make both S and T real) and general forms of mass matrices in models based on S_3 can be found in [23].

Now we describe a model based on a spontaneously broken S_3 flavour symmetry in which the lepton flavour basis is approximately reproduced by a particular VEV alignment. To achieve the desired alignment in a simple way, we work with a supersymmetric model, with $N=1$ SUSY, eventually broken by small soft breaking terms. The left-handed doublets transform as $(1 + 2)$ of S_3 and we will call $l_1 = (\nu_e, e)$ the invariant singlet and $D_l =$

$((\nu_\mu, \mu), (\nu_\tau, \tau))$ the S_3 doublet. The right-handed charged leptons e^c, μ^c, τ^c are all in the non-trivial singlet representation $1'$. The flavour symmetry is broken by two doublets φ_e, φ_ν and a singlet ξ . We assume that the flavon fields which break the S_3 symmetry are gauge singlets. In addition, we introduce an extra abelian symmetry Z_3 in such a way that φ_ν and ξ couple only to the neutrino sector and φ_e to the charged lepton sector, at the leading order. We summarize the transformation rules of the fields in Table 2.

Field	$h_{u,d}$	l_1	D_l	e^c	μ^c	τ^c	φ_e	φ_ν	ξ
S_3	1	1	2	$1'$	$1'$	$1'$	2	2	1
Z_3	1	ω	ω	ω^2	ω^2	ω^2	1	ω	ω

Table 2: Transformation properties of matter and flavon fields under the flavour group.

The Yukawa interactions for the lepton sector are controlled by the superpotential

$$w = w_\nu + w_e \quad (16)$$

where we have separated the contribution to neutrino masses and to charged lepton masses. For neutrinos we have:

$$w_\nu = \frac{h_u^2}{\Lambda^2} (y_1 D_l D_l \varphi_\nu + y_2 D_l D_l \xi + 2y_3 l_1 D_l \varphi_\nu + y_4 l_1 l_1 \xi) + \dots \quad (17)$$

where dots stand for higher-order contributions. As we will see later, φ_ν and ξ develop VEVs of the type

$$\langle \varphi_\nu \rangle = (v_\nu, v_\nu) \quad , \quad \langle \xi \rangle = u \quad . \quad (18)$$

After the flavour symmetry breaking and the electroweak symmetry breaking, w_ν becomes:

$$\begin{aligned} w_\nu &= \frac{h_u^2}{\Lambda^2} v_\nu (y_1 \nu_\mu \nu_\mu + y_1 \nu_\tau \nu_\tau + 2y_3 \nu_e \nu_\mu + 2y_3 \nu_e \nu_\tau) \\ &+ \frac{h_u^2}{\Lambda^2} u (2y_2 \nu_\mu \nu_\tau + y_4 \nu_e \nu_e) + \dots \end{aligned}$$

giving rise to the following neutrino mass matrix:

$$m_\nu = \frac{2h_u^2}{\Lambda^2} v_\nu \begin{pmatrix} y_4 x & y_3 & y_3 \\ y_3 & y_1 & y_2 x \\ y_3 & y_2 x & y_1 \end{pmatrix} \quad x \equiv \frac{u}{v_\nu} \quad . \quad (19)$$

This mass matrix is of the form (7) since w_ν respects explicitly the $\nu_\mu - \nu_\tau$ interchange symmetry. It is diagonalized by sending ν into $U_\nu \nu$, where

$$U_\nu = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & -1/\sqrt{2} \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad . \quad (20)$$

In the charged lepton sector, the leading terms of the superpotential are given by:

$$w_e = \gamma \tau^c (D_l \varphi_e)' \frac{h_d}{\Lambda} + (\beta' \mu^c + \gamma' \tau^c) (D_l \varphi_e \varphi_e)' \frac{h_d}{\Lambda^2} + (\alpha'' e^c + \beta'' \mu^c + \gamma'' \tau^c) l_1 (\varphi_e \varphi_e \varphi_e)' \frac{h_d}{\Lambda^3} + \dots \quad (21)$$

where dots stand for additional operators of order $1/\Lambda^3$, to be specified in the next section. The electroweak singlets e^c , μ^c and τ^c have the same quantum numbers and above, without loosing generality, we have defined τ^c as the field coupled to $(D_l \varphi_e)'$ and $(\beta' \mu^c + \gamma' \tau^c)$ as the combination coupled to $(D_l \varphi_e \varphi_e)'$. As we will see φ_e acquires a VEV of the type:

$$\langle \varphi_e \rangle = (v, 0) \quad . \quad (22)$$

Such a VEV breaks the parity symmetry generated by S in a maximal way, since

$$\langle \varphi_e \rangle^\dagger S \langle \varphi_e \rangle = 0 \quad . \quad (23)$$

We get:

$$w_e = \gamma \tau^c \tau v_d \left(\frac{v}{\Lambda} \right) + (\beta' \mu^c + \gamma' \tau^c) \mu v_d \left(\frac{v}{\Lambda} \right)^2 + (\alpha'' e^c + \beta'' \mu^c + \gamma'' \tau^c) e v_d \left(\frac{v}{\Lambda} \right)^3 \quad . \quad (24)$$

Notice that, provided the ratio v/Λ is much smaller than one, w_e generates a hierarchical mass pattern, as desired: $m_e \ll m_\mu \ll m_\tau$. We see that the ratio v/Λ should be approximately equal to the ratio m_μ/m_τ . We also notice that $(\varphi_e \varphi_e)' = 0$ due to anti-symmetry and consequently the electron mass is not generated at the same order as the muon mass⁵. This is a very nice result because we will end up with a nearly diagonal and hierarchical m_l without a Froggatt-Nielsen mechanism based on a continuous $U(1)$ flavour symmetry.

We define the expansion parameter

$$v/\Lambda \equiv \lambda^2 \ll 1 \quad . \quad (25)$$

The resulting charged lepton mass is of the type

$$m_l = \begin{pmatrix} \alpha'' \lambda^4 & 0 & 0 \\ \beta'' \lambda^4 & \beta' \lambda^2 & 0 \\ \gamma'' \lambda^4 & \gamma' \lambda^2 & \gamma \end{pmatrix} v_d \lambda^2 \quad , \quad (26)$$

with approximate eigenvalues:

$$m_\tau \approx \gamma \lambda^2 v_d \quad , \quad m_\mu \approx \beta' \lambda^4 v_d \quad , \quad m_e \approx \alpha'' \lambda^6 v_d \quad . \quad (27)$$

In order to reproduce correctly the charged lepton hierarchy, $\lambda \approx \lambda_c$ where λ_c is the Cabibbo angle. The adimensional constants γ , γ' , β' , γ'' , β'' , α'' are numbers with absolute value of order one and we find that the transformation needed to diagonalize m_l is:

$$V_e^T m_l U_e = \text{diag}(m_e, m_\mu, m_\tau) \quad , \quad (28)$$

⁵Observe that a term $(\alpha'' e^c + \beta'' \mu^c + \gamma'' \tau^c) l_1 (\varphi_\nu \varphi_\nu \varphi_\nu)' h_d/\Lambda$ would be allowed by S_3 and Z_3 symmetries but $(\varphi_\nu \varphi_\nu \varphi_\nu)' = 0$ due to the specific VEV alignment $\langle \varphi_\nu \rangle = v_\nu(1, 1)$. Similarly $(\varphi_\nu \varphi_\nu \xi)' = 0$

where the unitary matrix U_e , parametrized in the standard way, involves rotations of order $\theta_{12}^e = O(\lambda^2)$, $\theta_{13}^e = O(\lambda^4)$, $\theta_{23}^e = O(\lambda^2)$. The lepton mixing matrix is then $U_{PMNS} = U_e^\dagger U_\nu$ where U_ν is given by Eq. (20). We see that U_l introduces deviations of θ_{13} and θ_{23} from 0 and $\pi/4$, both of order λ^2 :

$$\theta_{13} = O(\lambda_c^2) \quad , \quad \theta_{23} = \frac{\pi}{4} + O(\lambda_c^2) \quad , \quad (29)$$

Additional deviations induced by sub-leading contributions will be discussed in the next section.

4 Vacuum alignment

In this section we discuss the minimization of the scalar potential leading to the results given in eqs. (18,22). Notice that the superpotential w of eqs. (16,17,21) is also invariant under a continuous $U(1)_R$ symmetry under which matter fields have $R = +1$, while Higgses and flavons have $R = 0$. Such a symmetry will be eventually broken down to R-parity by small SUSY breaking effects that can be neglected in first approximation in our analysis. The superpotential must have $R = 2$ and, to obtain non-trivial minima for the flavon fields, we introduce two driving multiplets with $R = 2$: a full singlet χ under the flavour group and a doublet ψ , transforming as $(2, \omega)$ under $S_3 \times Z_3$. Under these assumptions, at the leading order the superpotential depending on the driving fields is given by:

$$\begin{aligned} w_d &= a\chi\varphi_e^2 + b\psi\varphi_\nu^2 + c\psi\varphi_\nu\xi \\ &= 2a\chi\varphi_{e1}\varphi_{e2} + b\psi_1(\varphi_{\nu1})^2 + b\psi_2(\varphi_{\nu2})^2 + c\xi\psi_1\varphi_{\nu2} + c\xi\psi_2\varphi_{\nu1} \quad . \end{aligned} \quad (30)$$

In the SUSY limit the condition for the minima are:

$$\begin{aligned} \frac{\partial w_d}{\partial \chi} &= 2a\varphi_{e1}\varphi_{e2} = 0 \\ \frac{\partial w_d}{\partial \psi_1} &= b(\varphi_{\nu1})^2 + c\xi\varphi_{\nu2} = 0 \\ \frac{\partial w_d}{\partial \psi_2} &= b(\varphi_{\nu2})^2 + c\xi\varphi_{\nu1} = 0 \quad , \end{aligned} \quad (31)$$

This set of equations admit the solution:

$$\langle \varphi_e \rangle = (v, 0) \quad , \quad \langle \varphi_\nu \rangle = -\frac{c}{b}(u, u) \quad , \quad \langle \xi \rangle = u \quad , \quad (32)$$

with v and u arbitrary complex numbers. The flat directions along φ_{e1} and ξ can be removed by the interplay of radiative corrections to the scalar potential and soft SUSY breaking terms. As we have seen, to correctly reproduce the hierarchy of charged fermion masses we must assume $v/\Lambda = O(\lambda_c^2)$. The alignment of φ_e and φ_ν is modified when higher-dimensional operators are included in the analysis. At the next-to-leading order, the superpotential acquires the additional contribution

$$\delta w_d = \sum_{i=1}^8 \frac{d_i}{\Lambda} I_i \quad (33)$$

where I_i is a basis of independent invariants of dimension four:

$$\begin{aligned}
I_1 &= \chi\varphi_e^3 \\
I_2 &= \chi\varphi_\nu^3 \\
I_3 &= \chi\varphi_\nu^2\xi \\
I_4 &= \chi\xi^3 \\
I_5 &= (\psi\varphi_e)(\varphi_\nu^2) \\
I_6 &= (\psi\varphi_\nu)(\varphi_e\varphi_\nu) \\
I_7 &= \psi\varphi_\nu\varphi_e\xi \\
I_8 &= \psi\varphi_e\xi^2 \quad .
\end{aligned} \tag{34}$$

We look for a new SUSY minimum of the scalar potential generated by $w_d + \delta w_d$. By working around the leading-order minimum (32), we find

$$\langle\varphi_e\rangle = (v, \delta v) \quad , \quad \langle\varphi_\nu\rangle = -\frac{c}{b}(u + \delta u_1, u + \delta u_2) \quad , \quad \langle\xi\rangle = u \quad , \tag{35}$$

with v and u still arbitrary and

$$\begin{aligned}
\frac{\delta v}{v} &= \left[-\frac{d_1}{2a} + \frac{(2c^3d_2 - 2bc^2d_3 - b^3d_4)}{2ab^3} \left(\frac{u}{v}\right)^3 \right] \frac{v}{\Lambda} \\
\frac{\delta u_1}{u} &= \left[-\frac{c^2(2d_5 + 3d_6) - 2bcd_7 + b^2d_8}{3bc^2} \right] \frac{v}{\Lambda} \\
\frac{\delta u_2}{u} &= \left[-\frac{c^2(4d_5 + 3d_6) - 2bcd_7 + 2b^2d_8}{3bc^2} \right] \frac{v}{\Lambda} \quad .
\end{aligned} \tag{36}$$

We see that $\delta u_{1,2}/u$ are of order $v/\Lambda \approx \lambda_c^2$ and, barring accidental relations, they are different, thus modifying the leading order alignment $\varphi_\nu \propto (1, 1)$. We see that also $\delta v/v$ is of order $v/\Lambda \approx \lambda_c^2$, provided $u \leq v$. This last requirement is important in order to maintain the pattern of m_l discussed above.

The mass matrices m_ν , eq. (19), and m_l , eq. (26), are modified by subleading terms in the expansion in $1/\Lambda$. The neutrino mass matrix is modified by terms of relative order λ_c^2 by two independent effects: terms originating from higher-order operators containing an additional insertion of φ_e/Λ and terms originating from the modification to the leading order vacuum.

Coming to the charged lepton mass matrix, the higher dimensional operators whose effect cannot be absorbed in the parameters $\gamma, \gamma', \beta', \gamma'', \beta'', \alpha''$ are those contributing to the elements 12, 13 and 23, that vanish at the leading order. They are:

$$f^c D_l \varphi_\nu^3 \frac{h_d}{\Lambda^3} \quad , \quad f^c D_l \varphi_\nu^2 \xi \frac{h_d}{\Lambda^3} \quad , \quad f^c D_l \varphi_\nu \xi^2 \frac{h_d}{\Lambda^3} \tag{37}$$

where $f^c = (e^c, \mu^c, \tau^c)$. Their contribution is of order $(u/\Lambda)^3 \leq \lambda_c^6$ and they do not spoil our conclusion about the contribution U_e of m_l to the mixing matrix U_{PMNS} . Finally we should account for the modified vacuum, eq. (35). We find that also this effect does not modify the order of magnitude of the U_e mixing matrix. In conclusion, there are no further contribution to the lepton mixing coming from m_l and the modification to m_ν are of relative order λ_c^2 . Therefore our previous results, eq. (29), are stable under inclusion of subleading effects.

5 Quark masses and mixing angles

The flavour group $S_3 \times Z_3$, with few additional ingredients, can also provide a satisfactory description of the quark sector. We recall three special features of quarks. First, the top mass is very close to the electroweak symmetry breaking scale, $v = \sqrt{v_u^2 + v_d^2} \approx 174$ GeV, indicating an unsuppressed, renormalizable Yukawa coupling for the heaviest quark. Second, the hierarchy among quark masses of the up type is much more pronounced than the hierarchy in the down quark sector. Third, the mixing angle λ_c between the first two generations is the dominant one. The first feature suggests to adopt for quarks a different S_3 assignment. We still put left-handed quarks in a $(1+2)$ representation of S_3 , as for the left-handed leptons. We call $q_3 = (t, b)$ the invariant singlet and $D_q = ((u, d), (c, s))$ the S_3 doublet. At variance with the lepton sector, we assign all right-handed quarks to the invariant singlet 1^6 . As a consequence, there is one combination of right-handed quarks of up-type, denoted by t^c , directly coupled to a left-handed quark q_3 , the invariant singlet under S_3 , thus providing a renormalizable Yukawa interaction for the top quark. The second feature requires an extension of the flavour group, to differentiate up and down quarks. A simple extension is provided by an additional Z'_3 factor. The flavour symmetry becomes $S_3 \times Z_3 \times Z'_3$. Leptons do not transform under the new Z'_3 , so that the whole construction of the previous sections remains unchanged. In the quark sector we assume the transformation properties:

$$\begin{aligned} u^c &\rightarrow \omega u^c \\ (d^c, s^c, b^c, c^c) &\rightarrow \omega^2 (d^c, s^c, b^c, c^c) \end{aligned} \quad , \quad (38)$$

and the remaining fields are taken invariant under Z'_3 . To complete the construction we introduce a new flavon field, ξ' , transforming only under Z'_3 as $\xi' \rightarrow \omega \xi'$, and developing a large VEV:

$$\langle \xi' \rangle = u' \quad , \quad \frac{u'}{\Lambda} \equiv \lambda'^2. \quad (39)$$

These transformation properties are summarized in table 2. With this charge assignment,

Field	D_q	q_3	d^c, s^c, b^c	u^c	c^c	t^c	ξ'
S_3	2	1	1	1	1	1	1
Z_3	ω	ω	ω^2	ω^2	ω^2	ω^2	1
Z'_3	1	1	ω^2	ω	ω^2	1	ω

Table 3: Transformation properties of quarks and the flavon field ξ' under the flavour group.

the leading operators contributing to quark masses are:

$$w_q = w_{down} + w_{up} \quad . \quad (40)$$

⁶Observe that there is another equivalent choice of classification scheme under S_3 for quarks: q_i transform as $(1' + 2)$ and the right-handed quarks transform as $1'$. The resulting quark mass matrices remain unchanged.

In the down sector we have:

$$w_{down} = \gamma_d b^c q_3 \frac{\xi'}{\Lambda} h_d + (\beta'_d s^c + \gamma'_d b^c) (D_q \frac{\varphi_e}{\Lambda}) \frac{\xi'}{\Lambda} h_d + (\alpha''_d d^c + \beta''_d s^c + \gamma''_d b^c) (D_q \frac{\varphi_e \varphi_e}{\Lambda^2}) \frac{\xi'}{\Lambda} h_d + \dots \quad (41)$$

where we have defined b^c as the field coupled to q_3 and $(\beta'_d s^c + \gamma'_d b^c)$ as the combination coupled to $(D_q \varphi_e)$. For up type quarks we obtain:

$$\begin{aligned} w_{up} = & \left(\alpha_u \frac{\xi'^2}{\Lambda^2} u^c + \beta_u \frac{\xi'}{\Lambda} c^c + \gamma_u t^c \right) q_3 h_u \\ & + \left(\alpha'_u \frac{\xi'^2}{\Lambda^2} u^c + \beta'_u \frac{\xi'}{\Lambda} c^c + \gamma'_u t^c \right) (D_q \frac{\varphi_e}{\Lambda}) h_d \\ & + \left(\alpha''_u \frac{\xi'^2}{\Lambda^2} u^c + \beta''_u \frac{\xi'}{\Lambda} c^c + \gamma''_u t^c \right) (D_q \frac{\varphi_e \varphi_e}{\Lambda^2}) h_d + \dots \end{aligned} \quad (42)$$

The superpotential w_q , after electroweak and flavour symmetry breaking, gives rise to the following mass terms:

$$\begin{aligned} w_q = & \gamma_d b^c b v_d \lambda'^2 + (\beta'_d s^c + \gamma'_d b^c) s v_d \lambda^2 \lambda'^2 + (\alpha''_d d^c + \beta''_d s^c + \gamma''_d b^c) d v_d \lambda^4 \lambda'^2 \\ & + \left(\alpha_u \lambda'^4 u^c + \beta_u \lambda'^2 c^c + \gamma_u t^c \right) t v_u \\ & + \left(\alpha'_u \lambda'^4 u^c + \beta'_u \lambda'^2 c^c + \gamma'_u t^c \right) c v_u \lambda^2 \\ & + \left(\alpha''_u \lambda'^4 u^c + \beta''_u \lambda'^2 c^c + \gamma''_u t^c \right) u v_u \lambda^4 + \dots \end{aligned} \quad (43)$$

Assuming that the VEV of ξ' is similar to the VEV of ξ and φ_e :

$$\frac{u'}{\Lambda} \equiv \lambda'^2 \approx \frac{v}{\Lambda} \equiv \lambda^2 \quad , \quad (44)$$

we find that the quark mass matrices have the following pattern

$$m_d = \begin{pmatrix} \lambda^4 & 0 & 0 \\ \lambda^4 & \lambda^2 & 0 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_d \lambda^2 . \quad (45)$$

$$m_u = \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_u \quad , \quad (46)$$

where we have set to one the unknown coefficients $\alpha_{u,d}, \dots$. We see that the quark mass hierarchy is correctly reproduced by the interplay of the symmetry breaking parameters λ and λ' , once we take $\lambda \approx \lambda' \approx \lambda_c$.

$$\begin{aligned} m_t : m_c : m_u & \approx 1 : \lambda_c^4 : \lambda_c^8 \\ m_b : m_s : m_d & \approx 1 : \lambda_c^2 : \lambda_c^4 \end{aligned} .$$

We have sufficiently many order-one free parameters $\alpha_{u,d}, \dots$ to provide a good fit to the quark masses. Also the V_{CKM} mixing matrix is well reproduced, at least at the level

of the powers of λ_c , with the only exception of the Cabibbo angle, which in this model is a combination of two independent contributions of order λ_c^2 . We need an accidental enhancement of this combination in order to obtain the correct Cabibbo angle.

In summary, our flavour symmetry can be easily extended to the quark sector. Almost all quark masses and quark mixing angles are satisfactorily described by the spontaneous breaking of the symmetry $S_3 \times Z_3 \times Z'_3$. All dimensionless coefficients of the lagrangian are numbers of order one. No ad-hoc relations are required, with the exception of a moderate tuning needed to reproduce the Cabibbo angle. Unfortunately, the number of free parameters, the order-one coefficients $\alpha_{u,d}, \dots$, is too big to allow to formulate a quantitative, testable prediction, beyond the order-of-magnitude estimates illustrated above. A generic prediction of our model is that $\tan \beta = v_u/v_d$ is of the order one, since the hierarchy between top and bottom quark is ascribed to a symmetry breaking parameter, λ'^2 , rather than to $v_u \gg v_d$.

6 Conclusion

In the near future significant improvements on the parameters of the lepton mixing matrix are expected. The angles θ_{13} and θ_{23} will be constrained with an accuracy of about λ_c^2 , and it will be possible to establish more precisely how close are θ_{13} and θ_{23} to zero and to $\pi/4$, respectively. This will allow to discriminate between different models of fermion masses. Many models predict a generically large θ_{23} and a generically small θ_{13} , with deviations of order one from the reference values $\theta_{13} = 0$ and $\theta_{23} = \pi/4$. Only in a selected subset of the existing models a nearly maximal θ_{23} and a nearly vanishing θ_{13} are expected. Even in these last ‘special’ models $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ generally arise only as leading order results of a power series expansion. The expansion parameter is the ratio $\langle \varphi \rangle / \Lambda$ between the VEV of some field spontaneously breaking the underlying flavour symmetry, and the cut-off Λ of the theory. Therefore it is of great interest to provide an accurate estimate of the expected deviations from the leading order predictions. Here we have analyzed one of the simplest models that enforce $\theta_{13} \approx 0$ and $\theta_{23} \approx \pi/4$, thanks to an approximate $\nu_\mu - \nu_\tau$ parity symmetry of the neutrino sector. This parity symmetry is part of a larger flavour symmetry, $S_3 \times Z_3$, of the whole lepton sector, spontaneously broken along two different directions for neutrinos and charged leptons, respectively, as a consequence of a specific vacuum alignment. In the charged lepton sector the $\nu_\mu - \nu_\tau$ parity symmetry is maximally broken. A noticeable feature of the model is that the mass hierarchies between charged leptons are completely determined by the symmetry breaking of the permutation group, without the need of any additional ad-hoc ingredient. We find $\theta_{23} = \pi/4 + O(\lambda_c^2)$ and $\theta_{13} = O(\lambda_c^2)$, where the $O(\lambda_c^2)$ corrections come from the contribution of the charged lepton sector and are related to the mass hierarchies. We have carefully analyzed subleading effects due to higher dimensional operators that modify the Yukawa couplings and the vacuum alignment and we have verified that they do not spoil the above predictions. We have also briefly discussed how to extend the model to the quark sector, where, with a minimal enlargement of the flavour symmetry, masses and mixing angles are correctly reproduced, with the exception of the mixing angle between

the first two generations, that requires a small accidental enhancement.

Acknowledgements

We thank Guido Altarelli for useful discussion. We recognize that this work has been partly supported by the European Commission under contracts MRTN-CT-2004-503369 and MRTN-CT-2006-035505.

References

- [1] A. Strumia and F. Vissani, *Neutrino masses and mixings and...*, [arXiv:hep-ph/0606054]; S. Choubey, *What we can learn from atmospheric neutrinos*, [arXiv:hep-ph/0609182]; G. L. Fogli *et al.*, *Neutrino mass and mixing: 2006 status*, Nucl. Phys. Proc. Suppl. **168** (2007) 341; M. C. Gonzalez-Garcia and M. Maltoni, *Phenomenology with Massive Neutrinos*, arXiv:0704.1800 [hep-ph]; T. Schwetz, *Neutrino oscillations: present status and outlook*, arXiv:0710.5027 [hep-ph].
- [2] P. F. Harrison and W. G. Scott, *Permutation symmetry, tri-bimaximal neutrino mixing and the S_3 group characters*, Phys. Lett. B **557** (2003) 76 [arXiv:hep-ph/0302025]; E. Ma, *Aspects of the tetrahedral neutrino mass matrix*, Phys. Rev. D **72** (2005) 037301 [arXiv:hep-ph/0505209]; K. S. Babu and X. G. He, *Model of geometric neutrino mixing*, [arXiv:hep-ph/0507217]; W. Grimus and L. Lavoura, *A model realizing the Harrison-Perkins-Scott lepton mixing matrix*, JHEP **0601** (2006) 018 [arXiv:hep-ph/0509239]; E. Ma, *Tribimaximal neutrino mixing from a supersymmetric model with A_4 family symmetry*, Phys. Rev. D **73** (2006) 057304 [arXiv:hep-ph/0511133]; I. de Medeiros Varzielas and G. G. Ross, *$SU(3)$ family symmetry and neutrino bi-tri-maximal mixing*, Nucl. Phys. B **733** (2006) 31 [arXiv:hep-ph/0507176].
- [3] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions*, Nucl. Phys. B **720** (2005) 64 [arXiv:hep-ph/0504165]; G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing, $A(4)$ and the modular symmetry*, Nucl. Phys. B **741** (2006) 215 [arXiv:hep-ph/0512103]; G. Altarelli, F. Feruglio and Y. Lin, *Tri-bimaximal neutrino mixing from orbifolding*, Nucl. Phys. B **775** (2007) 31 [arXiv:hep-ph/0610165].
- [4] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, *Tri-bimaximal neutrino mixing and quark masses from a discrete flavour symmetry*, Nucl. Phys. B **775** (2007) 120 [arXiv:hep-ph/0702194].
- [5] For a review, see, for instance, G. Altarelli, *Lectures on Models of Neutrino Masses and Mixings* arXiv:0711.0161 [hep-ph] and references therein.

- [6] M. Ambrosio *et al.* [MACRO Collaboration], *Measurements of atmospheric muon neutrino oscillations, global analysis of the data collected with MACRO detector*, Eur. Phys. J. C **36** (2004) 323.
- [7] M. H. Ahn *et al.* [K2K Collaboration], *Measurement of neutrino oscillation by the K2K experiment*, Phys. Rev. D **74** (2006) 072003 [arXiv:hep-ex/0606032].
- [8] M. Kordosky, D. Petyt and f. t. M. Collaboration, *A Study of Muon Neutrino Disappearance Using the Fermilab Main Injector Neutrino Beam*, arXiv:0711.0769 [hep-ex].
- [9] Y. Ashie *et al.* [Super-Kamiokande Collaboration], *A measurement of atmospheric neutrino oscillation parameters by Super-Kamiokande I*, Phys. Rev. D **71** (2005) 112005 [arXiv:hep-ex/0501064].
- [10] C. W. Kim and U. W. Lee, *Comment on the possible electron-neutrino excess in the Super-Kamiokande atmospheric neutrino experiment*, Phys. Lett. B **444** (1998) 204 [arXiv:hep-ph/9809491]; O. L. G. Peres and A. Y. Smirnov, *Testing the solar neutrino conversion with atmospheric neutrinos*, Phys. Lett. B **456** (1999) 204 [arXiv:hep-ph/9902312].
- [11] M. Apollonio *et al.* [CHOOZ Collaboration], *Search for neutrino oscillations on a long base-line at the CHOOZ nuclear power station*, Eur. Phys. J. C **27** (2003) 331 [arXiv:hep-ex/0301017].
- [12] T. Fukuyama and H. Nishiura, *Mass matrix of Majorana neutrinos*, in *Proceedings of International Workshop on Masses and Mixings of Quarks and Leptons*, edited by Y. Koide (World Scientific, Singapore, 1997), [hep-ph/9702253];
E. Ma and M. Raidal, *Neutrino mass, muon anomalous magnetic moment, and lepton flavor nonconservation*, Phys. Rev. Lett. **87** (2001) 011802 [Erratum-ibid. **87** (2001) 159901] [arXiv:hep-ph/0102255]; C.S. Lam, *A 2-3 symmetry in neutrino oscillations*, Phys. Lett. B **507** (2001) 214 [hep-ph/0104116]; E. Ma, *The all-purpose neutrino mass matrix*, Phys. Rev. D **66** (2002) 117301 [hep-ph/0207352]; P.F. Harrison and W.G. Scott, *$\mu - \tau$ reflection symmetry in lepton mixing and neutrino oscillations*, Phys. Lett. B **547** (2002) 219 [hep-ph/0210197].
- [13] Y. Koide, *Universal texture of quark and lepton mass matrices with an extended flavor $2 \leftrightarrow 3$ symmetry*, Phys. Rev. D **69** (2004) 093001 [arXiv:hep-ph/0312207]; S. Morisi and M. Picariello, *The flavor physics in unified gauge theory from an $S(3) \times P$ discrete symmetry*, Int. J. Theor. Phys. **45** (2006) 1267 [arXiv:hep-ph/0505113]; A. S. Joshipura, *Universal 2-3 symmetry*, [arXiv:hep-ph/0512252]; K. Matsuda and H. Nishiura, *Broken flavor $2 \leftrightarrow 3$ symmetry and phenomenological approach for universal quark and lepton mass matrices*, Phys. Rev. D **73** (2006) 013008 [arXiv:hep-ph/0511338]; R. N. Mohapatra, S. Nasri and H. B. Yu, *Grand unification of mu-tau symmetry*, Phys. Lett. B **636** (2006) 114 [arXiv:hep-ph/0603020].

- [14] W. Grimus, A. S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka and M. Tanimoto, *Non-vanishing $U(e3)$ and $\cos(2\theta_{23})$ from a broken $Z(2)$ symmetry*, Nucl. Phys. B **713** (2005) 151 [arXiv:hep-ph/0408123].
- [15] A. de Gouvea, *Deviation of atmospheric mixing from maximal and structure in the leptonic flavor sector*, Phys. Rev. D **69** (2004) 093007 [arXiv:hep-ph/0401220]; C. S. Lam, *Neutrino 2-3 symmetry and inverted hierarchy*, Phys. Rev. D **71** (2005) 093001 [arXiv:hep-ph/0503159]; K. Fuki and M. Yasue, *What does μ - τ symmetry imply in neutrino mixings?* Phys. Rev. D **73** (2006) 055014 [arXiv:hep-ph/0601118], *Two categories of approximately μ - τ symmetric neutrino mass textures*, Nucl. Phys. B **783** (2007) 31 [arXiv:hep-ph/0608042]; R. N. Mohapatra, *θ_{13} as a probe of μ - τ symmetry for leptons*, JHEP **0410** (2004) 027 [arXiv:hep-ph/0408187]; N. Haba and W. Rodejohann, *A supersymmetric contribution to the neutrino mass matrix and breaking of μ - τ symmetry*, Phys. Rev. D **74** (2006) 017701 [arXiv:hep-ph/0603206].
- [16] T. Kitabayashi and M. Yasue, *μ - τ symmetry and maximal CP violation*, Phys. Lett. B **621** (2005) 133 [arXiv:hep-ph/0504212]; R. N. Mohapatra and W. Rodejohann, *Broken μ - τ symmetry and leptonic CP violation*, Phys. Rev. D **72** (2005) 053001 [arXiv:hep-ph/0507312]; T. Baba and M. Yasue, *Correlation between leptonic CP violation and μ - τ symmetry breaking*, Phys. Rev. D **75** (2007) 055001 [arXiv:hep-ph/0612034]; T. Baba and M. Yasue, *Leptonic CP Violation Induced by Approximately μ - τ Symmetric Seesaw Mechanism*, arXiv:0710.2713 [hep-ph].
- [17] W. Grimus and L. Lavoura, *Softly broken lepton number $L(e)$ - $L(\mu)$ - $L(\tau)$ with non-maximal solar neutrino mixing*, J. Phys. G **31** (2005) 683 [arXiv:hep-ph/0410279]; R. N. Mohapatra and S. Nasri, *Leptogenesis and μ - τ symmetry*, Phys. Rev. D **71** (2005) 033001 [arXiv:hep-ph/0410369]; R. N. Mohapatra, S. Nasri and H. B. Yu, *Leptogenesis, μ - τ symmetry and θ_{13}* , Phys. Lett. B **615** (2005) 231 [arXiv:hep-ph/0502026]; Y. H. Ahn, S. K. Kang, C. S. Kim and J. Lee, *Phased breaking of μ - τ symmetry and leptogenesis*, Phys. Rev. D **73** (2006) 093005 [arXiv:hep-ph/0602160]; Y. H. Ahn, C. S. Kim, S. K. Kang and J. Lee, *μ - τ symmetry and radiatively generated leptogenesis*, Phys. Rev. D **75** (2007) 013012 [arXiv:hep-ph/0610007].
- [18] C. S. Lam, *Symmetry of Lepton Mixing*, Phys. Lett. B **656** (2007) 193 [arXiv:0708.3665 [hep-ph]]; A. Blum, C. Hagedorn and M. Lindner, *Fermion Masses and Mixings from Dihedral Flavor Symmetries with Preserved Subgroups*, arXiv:0709.3450 [hep-ph]; A. Blum, C. Hagedorn and A. Hohenegger, *θ_C from the Dihedral Flavor Symmetries D_7 and D_{14}* , arXiv:0710.5061 [hep-ph].
- [19] C. D. Froggatt and H. B. Nielsen, *Hierarchy Of Quark Masses, Cabibbo Angles And CP Violation*, Nucl. Phys. B **147** (1979) 277.
- [20] S. F. King and G. G. Ross, *Fermion masses and mixing angles from $SU(3)$ family symmetry*, Phys. Lett. B **520** (2001) 243 [arXiv:hep-ph/0108112]; S. F. King and

- G. G. Ross, *Fermion masses and mixing angles from $SU(3)$ family symmetry and unification*, Phys. Lett. B **574** (2003) 239 [arXiv:hep-ph/0307190]; S. F. King, *Predicting neutrino parameters from $SO(3)$ family symmetry and quark-lepton unification*, JHEP **0508** (2005) 105 [arXiv:hep-ph/0506297]; I. de Medeiros Varzielas and G. G. Ross, *$SU(3)$ family symmetry and neutrino bi-tri-maximal mixing*, Nucl. Phys. B **733** (2006) 31 [arXiv:hep-ph/0507176].
- [21] E. Ma, *Permutation symmetry for neutrino and charged-lepton mass matrices*, Phys. Rev. D **61** (2000) 033012 [arXiv:hep-ph/9909249]; T. Kitabayashi and M. Yasue, *Large solar neutrino mixing and radiative neutrino mechanism*, Phys. Lett. B **524** (2002) 308 [arXiv:hep-ph/0110303]; T. Kitabayashi and M. Yasue, *$S(2L)$ permutation symmetry for left-handed mu and tau families and neutrino oscillations in an $SU(3)_L \times U(1)_N$ gauge model*, Phys. Rev. D **67** (2003) 015006 [arXiv:hep-ph/0209294]; I. Aizawa, M. Ishiguro, T. Kitabayashi and M. Yasue, *Bilarge neutrino mixing and mu - tau permutation symmetry for two-loop radiative mechanism*, Phys. Rev. D **70** (2004) 015011 [arXiv:hep-ph/0405201].
- [22] W. Grimus and L. Lavoura, *A discrete symmetry group for maximal atmospheric neutrino mixing*, Phys. Lett. B **572** (2003) 189 [arXiv:hep-ph/0305046]; W. Grimus and L. Lavoura, *Maximal atmospheric neutrino mixing and the small ratio of muon to tau mass*, J. Phys. G **30** (2004) 73 [arXiv:hep-ph/0309050]; W. Grimus and L. Lavoura, *$S(3) \times Z(2)$ model for neutrino mass matrices*, JHEP **0508** (2005) 013 [arXiv:hep-ph/0504153].
- [23] N. Haba and K. Yoshioka, *Discrete flavor symmetry, dynamical mass textures, and grand unification*, Nucl. Phys. B **739** (2006) 254 [arXiv:hep-ph/0511108].